Low Scale Theories: Light Neutrinos and Unification of Gauge Couplings

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Abstract

Mechanism for generation of suppressed neutrino masses, within low scale theories, is considered. The mechanism do not have extradimensional nature and is realized through extended $SU(2)_L$ scalar multiplets. Latters, in some cases, are also crucial for successful low scale unification.

1. *Introduction.* Despite great success in solving the gauge hierarchy problem, within low scale theories [1], there are problems and issues which should be reconsidered from a new viewpoint. One of the actual task is to understand the suppression of neutrino masses, which due to low fundamental scale, expected to be unacceptably large. In [2], for generation of suppressed neutrino masses, existence of extra dimensions have played crucial role. In [3], for the same purpose, together with right handed neutrinos was introduced additional scalar doublet with a sufficiently tiny VEV.

Here we demonstrate how the suppressed neutrino masses can be generated through the extended charged $SU(2)_L$ scalar multiplets [4]. Namely, 4, 5 or 6 dimensional plets should be applied respectively, depending on a value of fundamental scale M_f. These multiplets are crucial also for non SUSY low scale unification. For SUSY scenarios, some new possibilities of low scale unification are also found.

2. *Generation of Suppressed Neutrino Masses.* Introduce Φ scalar in $(4, \ -3)$ representation (REP) of $SU(2)_L\times U(1)_Y$. In this $U(1)_Y$ normalization, $Y(l)=1$ (l is lepton doublet). For avoiding $(lh^+)^2/M_f$ type operators (h is SM Higgs doublet), we assume that in fermion sector lepton number L is conserved. Prescribing to Φ lepton number -2, the Yukawa couplings responsible for neutrino masses possess $U(1)_L$ symmetry

$$\mathcal{L}_\nu=\hat{\lambda}_\nu ll\Phi h/M_f+h.c.\ , \tag{1}$$

where $\hat{\lambda}_\nu$ is matrix in a family space. For low M_f, scalar Φ should develop tiny VEV along its neutral component in order to generate suppressed neutrino masses. This is naturally insured through the scalar potential of h and Φ fields:

$$\mathcal{V}(h,\Phi)=\frac{\lambda_h}{2}\left(h^+h-m^2\right)^2+\frac{\lambda_\Phi}{2}\left(\Phi^+\Phi+M^2\right)^2+$$
$$\lambda_1\left(\Phi^+\Phi\right)\left(h^+h\right)+\lambda_2\left(\Phi^+h\right)\left(\Phi h^+\right)-\lambda\left(\Phi h^3+\Phi^+(h^+)^3\right)\ , \tag{2}$$

where m is Higgs doublet mass ~ 100 GeV. Last term in (2) mildly violates $U(1)_L$ and for all positive parameters in (2) system will have global minimum with non zero $\langle\Phi\rangle$. The extremum conditions for (2) will be:

$$\lambda_h(v^2-m^2)+(\lambda_1+\lambda_2)V^2-3\lambda Vv=0\ ,$$
$$\lambda_\Phi(V^2+M^2)V+(\lambda_1+\lambda_2)Vv^2-\lambda v^3\ =0\ , \tag{3}$$

and for $\lambda_\Phi M^2\gg(\lambda_1+\lambda_2)m^2$, one can easily obtain

$$v=m+\mathcal{O}\left(m^3/M^2\right)\ ,\qquad V=\lambda v^3/(\lambda_\Phi M^2)+\mathcal{O}\left(m^5/M^4\right)\ . \tag{4}$$

Note, that although the mass of Φ is much larger than v, the hierarchy is not destabilized, because Φ's VEV in (4) is tiny and quartic terms in (2) practically do not affect v. Using (4) in (1), for neutrino masses we will have

$$\hat{m}_\nu=\hat{\lambda}_\nu vV/M_f\simeq\lambda\hat{\lambda}_\nu v^4/(\lambda_\Phi M^2M_f), \tag{5}$$

and desirable value $\hat{m}_\nu=(1-4\cdot 10^{-2})$ eV is obtained for $M\simeq M_f=(1-3)\cdot 10^3$ TeV with $v=174$ GeV, $\lambda\hat{\lambda}_\nu/\lambda_\Phi\sim 1$.

If we wish to build scenario with lower M_f, higher $SU(2)_L$ REPs must be introduced. Namely, if now Φ is 5-plet of $SU(2)_L$ with $Y(\Phi)=-4$, then instead of (1) we will have $\mathcal{L}_\nu=\hat{\lambda}_\nu ll\Phi h^2/M_f^2+\text{h.c.}$, and in potential (2) last term will be replaced with $-\lambda'(\Phi h^4+\Phi^+h^{+4})/M_f$. For this case it is easy to verify that $v\simeq m$, $V\simeq\lambda' v^4/(\lambda_\Phi M^2M_f)$ and consequently for neutrino masses

$$\hat{m}_\nu=\hat{\lambda}_\nu v^2V/M_f^2\simeq\lambda'\hat{\lambda}_\nu v^6/(\lambda_\Phi M^2M_f^3)\ , \tag{6}$$

which for $\hat{m}_\nu=(1-0.1)$ eV, $\lambda'\hat{\lambda}_\nu/\lambda_\Phi\sim 1$ require relatively low scales $M\simeq M_f=(30-50)$ TeV.

Scales M_f, M can be easily reduced even down to few TeV, if Φ belongs to $(6,\ -5)$ REP of $SU(2)_L\times U(1)_Y$. Then instead the last term in (2) we will have $-\lambda''(\Phi h^5+\Phi^+h^{+5})/M_f^2$ and relevant Yukawa couplings will be $\hat{\lambda}_\nu ll\Phi h^3/M_f^3$. By simple analyses one can easily obtain that in this case

$$\hat{m}_\nu\simeq\lambda''\hat{\lambda}_\nu v^8/(\lambda_\Phi M^2M_f^5)\ , \tag{7}$$

and $(1-0.1)$ eV neutrino masses are generated for $M\simeq M_f=(7-10)$ TeV.

Supersymmetrizing these scenarios, together with superfield Φ (which denote 4, 5 or 6-plets) we introduce its conjugate superfield $\overline{\Phi}$. Relevant superpotential is

$$W_\Phi=M\overline{\Phi}\Phi-\left(\lambda_{\Phi d}\Phi h_d^{3+n}+\lambda_{\Phi u}\overline{\Phi}h_u^{3+n}\right)/M_f^{1+n}\ , \tag{8}$$

where $n = 0, 1, 2$ for scenarios with $\Phi + \overline{\Phi}$ in 4, 5 and 6 REPs of $SU(2)_L$ respectively. h_u, h_d denote doublet-untidoublet pair of MSSM and $\lambda_{\Phi d}$, $\lambda_{\Phi u}$ are positive dimensionless couplings of the order of one. Yukawa superpotential, responsible for neutrino masses, will be

$$W_\nu = \hat{\lambda}_\nu l l \Phi h_d^{n+1}/M_f^{n+1} \ . \tag{9}$$

After that SUSY and EW symmetry breaking take place, non zero $\langle h_u \rangle$, $\langle h_d \rangle$ are generated and from (8) one can easily verify $\langle \Phi \rangle \simeq \lambda_{\Phi u} \langle h_u \rangle^{n+3}/(M M_f^{n+1})$. Using this and also (9), we will get

$$\hat{m}_\nu = \hat{\lambda}_\nu \lambda_{\Phi u} \sin^{n+3}\beta \cos^{n+1}\beta \cdot v^{2n+4}/(M M_f^{2n+2}) \ , \tag{10}$$

where we have used $\langle h_u \rangle = v \sin\beta$, $\langle h_d \rangle = v \cos\beta$. For $v = 174$ GeV, $\tan\beta \simeq 1$, neutrino masses $m_\nu \sim (1 - 0.1)$ eV are obtained within various scenarios:

$$M \simeq M_f = \begin{cases} (0.6 - 1.3) \cdot 10^3 \ \text{TeV}; & n = 0, \ \text{case with } 4-\text{plets} \\ (20 - 30) \ \text{TeV}; & n = 1, \ \text{case with } 5-\text{plets} \\ (4.7 - 6.5) \ \text{TeV}; & n = 2, \ \text{case with } 6-\text{plets} \end{cases} \ . \tag{11}$$

Larger values of $\tan\beta$ would give stronger suppression for $\hat{m}_\nu$ in (10), giving possibility to reduce mass scales in (11) by few factors.

Mechanisms which we have suggested here, provide adequate suppressions of neutrino masses and this suppressions occur through proper choice of Φ scalar in appropriate $SU(2)_L \times U(1)_Y$ REP. Neutrino mass scale $\sim (0.1 - 1)$ eV is natural for atmospheric anomaly if three family neutrinos are either hierarchical in mass or degenerate, respectively. For simultaneous accommodation of atmospheric and solar neutrino data, one can introduce flavor symmetries and build different oscillation scenarios in a spirit of [5].

3. *Gauge Coupling Unification.* Due to extra spacelike dimensions (with radius R), gauge couplings can get power low runnings starting from scale $\mu_o = 1/R$. This gives possibility for low scale unification [6]. Solutions of 1-loop RGEs are

$$\alpha_G^{-1} = \alpha_a^{-1}(M_Z) - \frac{b_a}{2\pi} \ln \frac{M_G}{M_Z} - \frac{\tilde{b}_a^i}{2\pi} \ln \frac{M_G}{M_i} - \frac{\hat{b}_a^i}{2\pi} P_\delta^{(\mu_i)} \ , \tag{12}$$

where $\alpha_{1,2,3}$ denote gauge couplings of $U(1)$, $SU(2)_L$ and $SU(3)_c$ respectively, b_a is b-factors of SM/MSSM, $\tilde{b}_a^i$ come from contribution of additional states of mass $M_i (< M_G$ GUT scale$\simeq M_f)$, $\hat{b}_a^i$ come from Kaluza-Klein (KK) states;

$$P_\delta^{(\mu_i)} = \frac{X_\delta}{\delta} \left[(M_G/\mu_i)^\delta - 1 \right] - \ln M_G/\mu_i \ , \quad X_\delta = \frac{2\pi^{\delta/2}}{\delta \Gamma(\delta/2)} \ , \quad \mu_i^2 = M_i^2 + \mu_0^2 \ . \tag{13}$$

From (12), (13) one can see that for various scenarios successful unification with $\alpha_s \simeq 0.119$ is achieved for: **a)** Non SUSY scenario with two $\Phi(4)$-plets and values of extra dimensions and scales:

$$(\delta\ ,\ M_G/\mu_0\ ,\ M_G/M\ ,\ M_G) = (1, \quad 9.78, \quad 6.51, \quad 10^{3.51}\ \mathrm{TeV})\ ,$$
$$(2, \quad 3.45, \quad 2.83, \quad 10^{3.26}\ \mathrm{TeV})\ ,\ (3, \quad 2.36, \quad 2.071, \quad 10^{3.18}\ \mathrm{TeV})\ , \cdots \tag{14}$$

b) non SUSY case with one $\Phi(5)$-plet and

$$(\delta\ ,\ M_G/\mu_0\ ,\ M_G/M\ ,\ M_G) = (1, \quad 11.55, \quad 6.9, \quad 10^{1.82}\ \mathrm{TeV})\ ,$$
$$(2, \quad 3.735, \quad 2.92, \quad 10^{1.67}\ \mathrm{TeV})\ ,\ (3, \quad 2.486, \quad 2.12, \quad 10^{1.61}\ \mathrm{TeV})\ , \cdots \tag{15}$$

c) non SUSY case with one $\Phi(6)$-plet and

$$(\delta\ ,\ M_G/\mu_0\ ,\ M_G/M\ ,\ M_G) = (1, \quad 12.25, \quad 4.55, \quad 10.7\ \mathrm{TeV})\ ,$$
$$(2, \quad 3.837, \quad 2.35, \quad 8.69\ \mathrm{TeV})\ ,\ (3, \quad 2.486\ ,\ 2.12, \quad 8.11\ \mathrm{TeV})\ , \cdots \tag{16}$$

For **a)-c)** cases Φ-plets are crucial for unification. Number of chiral families, with KK excitations, can be $\eta = 0 \div 3$. For all this η, the α_G remain perturbative. SUSY unification require one additional $SU(3)_c$ adjoint state (for each scenario) with KK excitations and without zero mode wave function. Consequently, there are different cases of unification ($\alpha_s = 0.119$): **d)** SUSY scenario with one pair of $\Phi(4) + \overline{\Phi}(\overline{4})$ supermultiplets and

$$(\delta\ ,\ M_G/\mu_0\ ,\ M_G/M\ ,\ M_G) = (1, \quad 18.57, \quad 10.56, \quad 10^{3.13}\ \mathrm{TeV})\ ,$$
$$(2, \quad 4.78, \quad 3.657, \quad 10^{2.96}\ \mathrm{TeV})\ ,\ (3, \quad 2.937, \quad 2.465, \quad 10^{2.92}\ \mathrm{TeV})\ , \cdots \tag{17}$$

e) SUSY scenario with one pair of $\Phi(5) + \overline{\Phi}(\overline{5})$ supermultiplets and

$$(\delta\ ,\ M_G/\mu_0\ ,\ M_G/M\ ,\ M_G) = (1, \quad 18.02, \quad 6.09, \quad 10^{1.46}\ \mathrm{TeV})\ ,$$
$$(2, \quad 4.683, \quad 2.742, \quad 10^{1.39}\ \mathrm{TeV})\ ,\ (3, \quad 2.895, \quad 2.029, \quad 10^{1.36}\ \mathrm{TeV})\ , \cdots \tag{18}$$

f) SUSY scenario with one pair of $\Phi(6) + \overline{\Phi}(\overline{6})$ supermultiplets and

$$(\delta\ ,\ M_G/\mu_0\ ,\ M_G/M\ ,\ M_G) = (1, \quad 16.84, \quad 3.9, \quad 5.74\ \mathrm{TeV})\ ,$$
$$(2, \quad 4.515, \quad 2.169, \quad 5.25\ \mathrm{TeV})\ ,\ (3, \quad 2.823, \quad 1.729, \quad 5.12\ \mathrm{TeV})\ , \cdots \tag{19}$$

In cases **d)-f)** only $\eta = 0$ is allowed. For higher values of η gauge couplings become non perturbative. Through analyses, we have taken $M_f \simeq M_G$. It is possible to have M_G, by few factors and even more, below the M_f. This would reduce scales μ_0, M, making scenarios easily testable on a future colliders.

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